# Solar collector performance

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*In this investigation, the temperatures of the insulated collector plate and glass cover of a horizontal solar collector were measured and compared to theoretically predicted values for different ambient conditions. By employing an appropriate equation for the prediction of the heat transfer between the cover and the natural environment, good agreement was obtained between the theoretically predicted and experimentally measured values.*

## Nomenclature

| | |
|---|---|
| $C_e$ | Coefficient of extinction, $m^{-1}$ |
| $c_p$ | Specific heat capacity of air, J/kg/K |
| $DOY$ | Day of the year |
| $EOT$ | Equation of time, minutes |
| $Gr_{ca}$ | Grashof number, $2(T_p - T_a)gL^3\rho^2/((T_p+T_a)\mu^2)$ |
| $h$ | Convective heat transfer coefficient, $W/m^2K$ |
| $I_h$ | Solar radiation on horizontal surface, $W/m^2$ |
| $k$ | Thermal conductivity of air, W/mK |
| $L_c$ | Length of collector side, m |
| $n$ | Refractive index |
| $p_a$ | Atmospheric pressure, $N/m^2$ |
| Pr | Prandtl number, $\mu c_p/k$ |
| $q$ | Heat flux, $W/m^2$ |
| $q_c$ | Convection heat transfer rate, $W/m^2$ |
| $q_r$ | Radiation heat transfer rate, $W/m^2$ |
| $Ra_{pc}$ | Plate-cover Rayleigh number $2(T_p - T_a)g{t_a}^3{\rho_{pc}}^2c_{ppc}/((T_p+T_a)k_{pc}\mu_{pc})$ |
| Re | Reynolds number, $\rho v_w L/\mu$ |
| $t$ | Thickness, m |
| $T$ | Temperature, °C or K |
| $v_w$ | Wind speed, m/s |
| $Y$ | Year |
| $YADJ$ | Leap year adjustment, days |

*Greek Letters*

| | |
|---|---|
| $\alpha_p$ | Absorption coefficient of plate |
| $\phi_d$ | Declination angle, ° |
| $\phi_1$ | Latitude angle of location, ° |
| $\phi_{long}$ | Longitude angle of location, ° |
| $\phi_{longs}$ | Standard meridian for time zone, ° |
| $\phi_p$ | Annual phase angle, radians |
| $\theta$ | Incidence angle, ° |
| $\rho$ | Density of air, $kg/m^3$ |
| $\varepsilon_c$ | Emissivity of cover |
| $\varepsilon_p$ | Emissivity of plate |
| $\tau_a$ | Transmittance due to absorptance |
| $\mu$ | Dynamic viscosity of air, kg/m s |
| $\sigma$ | Stefan-Boltzman constant, $5.67 \times 10^{-8} W/m^2K^4$ |
| $\omega$ | Hour angle, ° |
| $\psi$ | Time after midnight, h |

*Subscripts*

| | |
|---|---|
| $a$ | Atmospheric |
| $b$ | Beam |
| $c$ | Cover |
| $ca$ | Evaluate at mean temperature between cover and ambient air |
| $d$ | Diffuse |
| $p$ | Plate |
| $pc$ | Evaluate at mean temperature between plate and cover |
| $sky$ | Sky |
| $w$ | Wind |

## Introduction

It is interesting to note that although excellent books and a mass of literature on the theory concerning the performance of all types of solar collectors are available, no reference could be found of the simplest horizontal collector, consisting of an insulated horizontal square plate covered by a single glass sheet, which is supported by insulated side walls, where the experimentally measured plate surface and cover temperatures were compared to theoretically predicted values for a particular solar radiation and other given ambient conditions.

If an analysis had been conducted and the calculated plate temperature had been in agreement with the measured value, this would have been fortuitous since no reliable equation for the prediction of the heat transfer rate between the cover and the natural environment was, until recently, available to make such a prediction possible.

Recent work[1] presented an equation with which the effective heat transfer (both convection and radiation) from a glass cover to the ambience can be predicted for a particular collector geometry (similar to the present collector), i.e.

$$q = \sigma\varepsilon_c\left[T_c^4 - \left(0.0552T_a^{1.5}\right)^4\right] + k_{ca}\left(Gr_{ca}\,\mathrm{Pr}_{ca}\right)^{1/3}$$
$$\times\left(0.227 + 1.406\times10^{-6}\,\mathrm{Re}_{ca}\right)\left(T_c - T_a\right)/L_c$$

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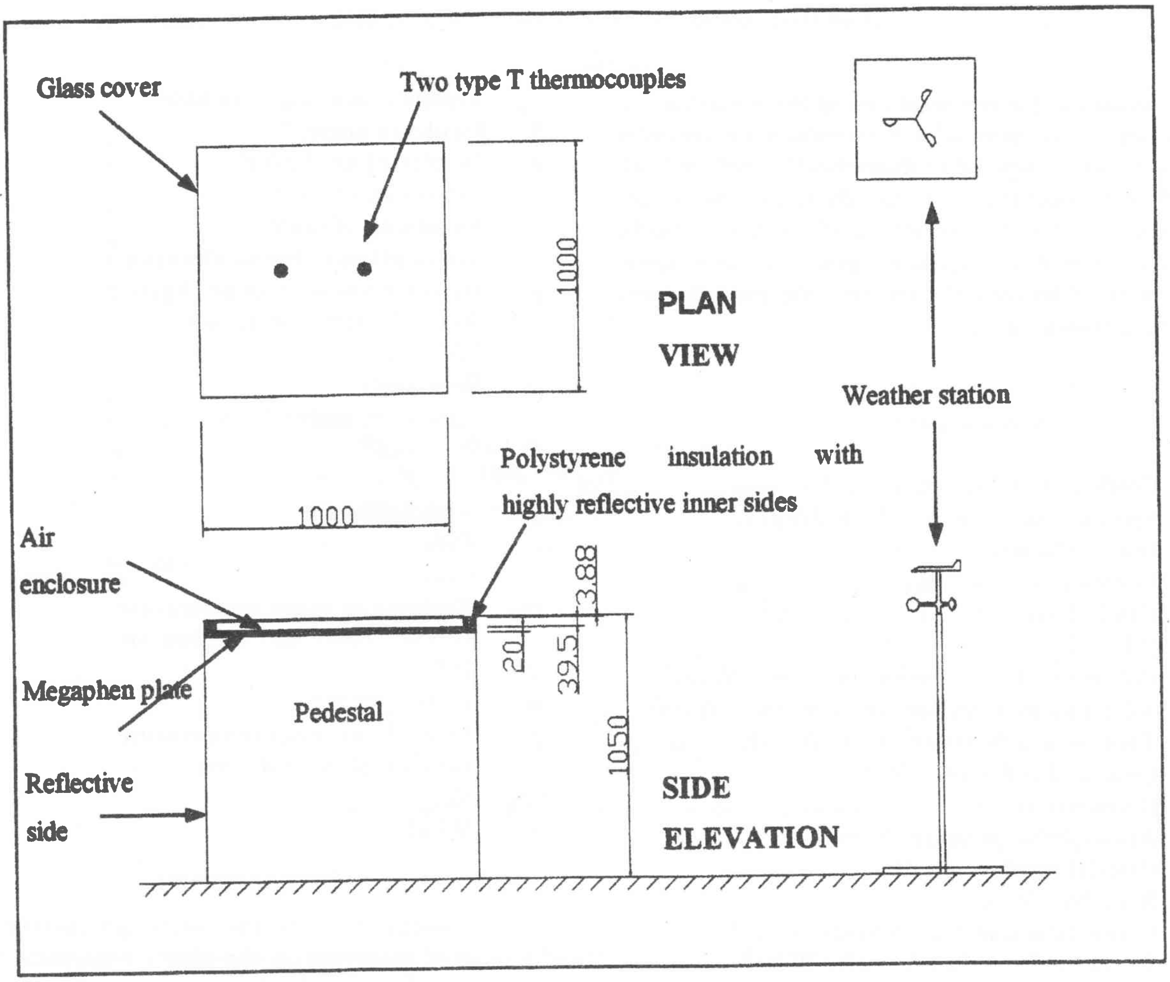


Fig 1. Solar collector apparatus

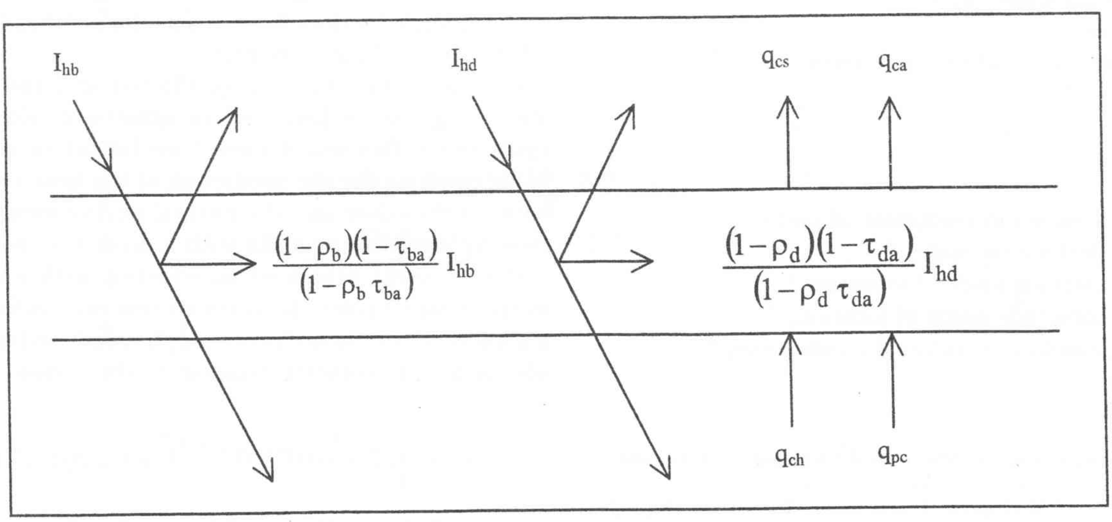


Fig. 2. Glass cover energy balance

In this equation it was assumed that the Kelvin sky temperature is given approximately by $T_{sky} = 0.0552T_a^{1.5}$. This relation was considered to be reasonably accurate under relatively dry clear sky conditions. Since we were interested in an accurate prediction of the effective heat transfer from the glass surface and not the individual effects, i.e. convective heat transfer (natural and forced) and radiation heat transfer, uncertainties in the individual terms of the above equation were not critical (the combination of the terms is important).

In this investigation the glass cover and plate temperatures of a simple solar collector were measured and were found to be in good agreement with theoretically predicted values.

## Apparatus

The experimental solar collector, as shown in Fig. 1, consisted of a horizontal 1 m × 1 m square glass cover with a thickness of 3.88 mm. The base of the solar collector consisted of a 20 mm thick insulated (megaphen with $k_p = 0.015$ W/mK) black plate ($\alpha_p = 0.9$) located 39.5 mm below the glass cover. The sides of the collector consisted of 50 mm thick polystyrene sheets with the inside surfaces covered by a highly reflective layer of aluminium foil ($\alpha \approx \varepsilon = 0.1$). A square pedestal supported the collector having highly reflective sides, at an elevation of 1.05 m above ground level. Plate and cover temperatures were measured with the aid of type-T thermocouples (two thermocouples on each surface). The ambient conditions (temperature and wind) were obtained from a Davis weather station located near the apparatus as shown in Fig. 1, while the total incident solar radiation was measured with the aid of a Kipp and Zonen solar sensor placed on top of the glass cover. The test facility was located 100 m above sea level at 33.98°S latitude and 18.85°E longitude.

## Analysis

The objective of this study was to compare the experimentally measured glass cover and plate temperatures of a solar collector with theoretically predicted values for different ambient conditions.

To analyse the problem, consider the solar collector (shown in Fig. 1) that was subjected to a solar radiation of

$$I_h = I_{hb} + I_{hd} \tag{1}$$

where $I_{hb}$ and $I_{hd}$ are the beam and diffuse solar radiation on a horizontal surface, respectively.

A part of the beam radiation is reflected at the collector cover (Duffie and Beckman[2]). For a refraction index of $n = 1.526$ the reflection at the upper surface of the glass cover is

$$\rho_b = \left[\frac{\sin^2\{\arcsin(\sin\theta/1.526) - \theta\}}{\sin^2\{\arcsin(\sin\theta/1.526 + \theta)\}}\right] + \frac{1}{2}\left[\frac{\tan^2\{\arcsin(\sin\theta/1.526) - \theta\}}{\tan^2\{\arcsin(\sin\theta/1.526) + \theta\}}\right] \tag{2}$$

where the zenith or beam incidence angle $\theta$ is determined from the relation

$$\cos\theta = \sin\phi_d \sin\phi_1 + \cos\phi_d \cos\phi_1 \cos\omega \tag{3}$$

where

$\phi_d =$ declination i.e. the angular position of the sun at solar noon with respect to the plane of the equator (North positive), degrees

$\phi_1 =$ latitude (North positive), degrees

$\omega =$ hour angle degrees, solar noon being zero and each hour equalling 15° of longitude with mornings positive and afternoons negative, e.g. $\omega = +15°$ for 11h00 and $-37.5°$ for 14h30.

The declination angle is given by

$$\phi_d = 180\left[\begin{array}{l} 0.00661 + 0.40602\sin(\phi_p - 1.4075) \\ +0.00665\sin(2\phi_p - 1.4789) \\ +0.00298\sin(3\phi_p - 1.0996) \end{array}\right]/\pi \tag{4}$$

where $\phi_p$ is the annual phase angle given by

$$\phi_p = 0.0172028\,(DOY + YADJ + 0.417) \tag{5}$$

In equation (5) $DOY$ is the day of the year, where 1 January is the first day of the year and $YADJ$ is the leap year adjustment which is given by

$$YADJ = 0.25\,[2.5 - \{Y - 4\,(\text{integer}\,[(Y-1)/4])\}] \tag{6}$$

where $Y$ is the particular year. The integer function rounds off the calculated value towards the lower integer value.

The hour angle $\omega$ is determined according to the following equation

$$\omega = 15\left[\psi - 12 - \{4\,(\phi_{longs} - \phi_{long}) - EOT\}/60\right] \tag{7}$$

The equation of time ($EOT$), expressed in minutes, is given by

$$EOT = 1440\,[0.005114\sin(\phi_p + 3.0593)] \\ +0.006892\sin(2\phi_p + 3.4646) \\ +0.000222\sin(3\phi_p + 3.3858) \\ +0.000153\sin(4\phi_p + 3.7766) \tag{8}$$

For diffuse radiation, employ equation (2) with $\theta = 60°$,[2] to find the reflectivity $\rho_d = 0.09346$. The beam transmittance considering only absorption through the glass cover is given by

$$\tau_{ba} = e^{-C_e t_c/\cos\{\arcsin(\sin\theta/1.526)\}} \tag{9}$$

where $C_e$. is the extinction coefficient and $t_c$ is the thickness of the glass cover.

The transmittance of diffuse radiation considering only absorptance, $\tau_{da}$, is the same as that for $\tau_{ba}$ at $\theta = 60°$.

Ultimately the fraction of solar beam radiation absorbed by the plate is given by[2,3]

$$(\tau_b\alpha_p)_b = (1-\rho_b)^2\,\tau_{ba}\alpha_p/\left[\{1-\rho_d(1-\alpha_p)\}\left(1-\rho_b^2\tau_{ba}^2\right)\right] \tag{10}$$

where $\alpha_p$ is the solar absorptance of the plate.

Similarly the fraction of diffuse radiation absorbed by the plate is

$$(\tau_d\alpha_p)_d = (1-\rho_b)^2\,\tau_{da}\alpha_p/\left[\{1-\rho_d(1-\alpha_p)\}\left(1-\rho_d^2\tau_{da}^2\right)\right]$$

Rearranging the previous equation, the absorbed plate diffuse radiation becomes

$$(\tau_d\alpha_p)_d = 0.8218\tau_{da}\alpha_p \,/\left[(0.90654+0.09346\alpha_p)\left(1-0.00873\tau_{da}^2\right)\right] \tag{11}$$

The net solar radiation per unit area absorbed by the plate is therefore

$$q_p = (\tau_b\alpha_p)_b\, I_{hb} + (\tau_d\alpha_p)_d\, I_{hd} \tag{12}$$

Since the base plate and the sides of the collector were well insulated, conduction losses can be neglected. Radiative exchange with the highly reflective sides of the collector can also be ignored.

With these assumptions the energy balance applicable to the surface of the plate is

$$q_p = q_{ph} + q_{pc} \tag{13}$$

where the convective heat transfer from the plate to the air is

$$q_{ph} = h_{pc}(T_p - T_c) \tag{14}$$

If the plate is the warmer of the two surfaces, the convective heat transfer from the air is given by[4]

$$h_{pc} = \left[\begin{array}{c} 1+1.44\,(1-1708/Ra_{pc}) \\ +\left\{(Ra_{pc}/5830)^{1/3}-1\right\}\end{array}\right] k_{pc}/t_a \tag{15}$$

where $k_{pc}$ is the thermal conductivity of the air at $(T_p + T_c)/2$ and $t_a$ is the distance between cover and the plate. This equation is valid for Rayleigh numbers ranging from 0 to $10^8$. The Rayleigh number is

$$Ra_{pc} = 2(T_p - T_c)\, g t_a^3 \rho_{pc}^2 c_{ppc} \,/\left[(T_p+T_c)\,\mu_{pc}k_{pc}\right] \tag{16}$$

where all the air properties are evaluated at the mean air temperatures $(T_p + T_c)/2$.

The radiative heat exchange between the plate and cover is given by

$$q_{pc} = \sigma\left(T_p^4 - T_c^4\right)/\left(1/\varepsilon_p + 1/\varepsilon_c + 1\right) \tag{17}$$

The steady state energy (this was essentially the case at solar noon) applicable to the glass cover control volume, shown in Fig. 2, is

$$\frac{(1-\rho_b)(1-\tau_{ba})}{(1-\rho_b\tau_{ba})}I_{hb} + \frac{(1-\rho_d)(1-\tau_{da})}{(1-\rho_d\tau_{da})}I_{hd} + q_{ch} + q_{pc} = (q_{cs}+q_{ca}) = q \tag{18}$$

where temperature gradients in the glass are neglected.

For steady state conditions (see Fig. 2) the convective heat transfer from the air below the cover to the cover is the same as the convective heat transfer from the plate to the air, i.e.

$$q_{ch} = q_{ph} = h_{pc}(T_p - T_c) \tag{19}$$

The heat loss from the upper surface of this particular collector geometry to the ambience is given by[1]

$$(q_{cs}+q_{ca}) = q = \sigma\varepsilon_c\left[T_c^4 - \left(0.0552T_a^{1.5}\right)^4\right] + \left[(Gr_{ca}\,\mathrm{Pr}_{ca})^{1/3}\left(0.227 + 1.406\times 10^{-6}Ra_{ca}\right)\right] \times k_{ca}(T_c - T_a)/L_c \tag{20}$$

By solving equations (13) and (18) simultaneously, the cover and plate temperatures of the solar collector can be predicted for given ambient conditions.

## Results and Conclusion

The results of measurements conducted during midday when essentially steady state conditions prevail are shown in Figs. 3 and 4. A numerical example, in which cover and plate temperatures are predicted theoretically at a time during this period, is presented in the Appendix.

The agreement between the measured temperatures ($T_c$ and $T_p$) and the predicted values is very good. This suggests that the theoretical approach and the equations used make it possible to predict the performance of this type of collector to a high degree of accuracy.

## References


[1] Lombaard IF & Kroger DG. Heat transfer between a horizontal flat surface and the natural environment. *R&D Journal*, 2001, **17**, pp.47–52.

[2] Duffie JA & Beckman WA. *Solar Engineering of Thermal Processes*, 2nd edn. Wiley Interscience, New York, 1991.

[3] Modest MF. *Radiative Heat Transfer*, 3rd edn. New York, 1993.

[4] Hollands KGT, Unny TE, Raithby GG & Konicek L. Free convection heat transfer across inclined air layers. *J. of Heat Transfer*, 1976, **98**, pp.189–193.


## Appendix: Numerical Example

A solar collector test was conducted at 12h39:44 ($\psi$ = 12.6622 hours after midnight) local time on 27 October 2000 ($DOY$ = 271st day of the year) at the Solar Energy Laboratory of the University of Stellenbosch, South Africa, located on latitude $\phi_1 = -33.98°$ (South) and $\phi_{long} = 18.85°$ (East). The South African standard meridian is $\phi_{longs} = 30°$ (East).

The collector specifications are as follows (see Fig. 1):

| | |
|---|---|
| Length of collector side (square) | $L_c = 1$ m |
| Glass cover thickness | $t_c = 3.88$ mm |
| Height of air layer between plate and cover | $t_{air} = 39.5$ mm |
| Long-wave emissivity of glass | $\varepsilon_c = 0.88$ |
| Extinction coefficient of glass | $C_e = 13$ m$^{-1}$ |
| Refractive index of glass | $n_c = 1.526$ |
| Long-wave emissivity of plate | $\varepsilon_p = 0.9$ |
| Solar absorptivity of plate | $\alpha_p = 0.9$ |

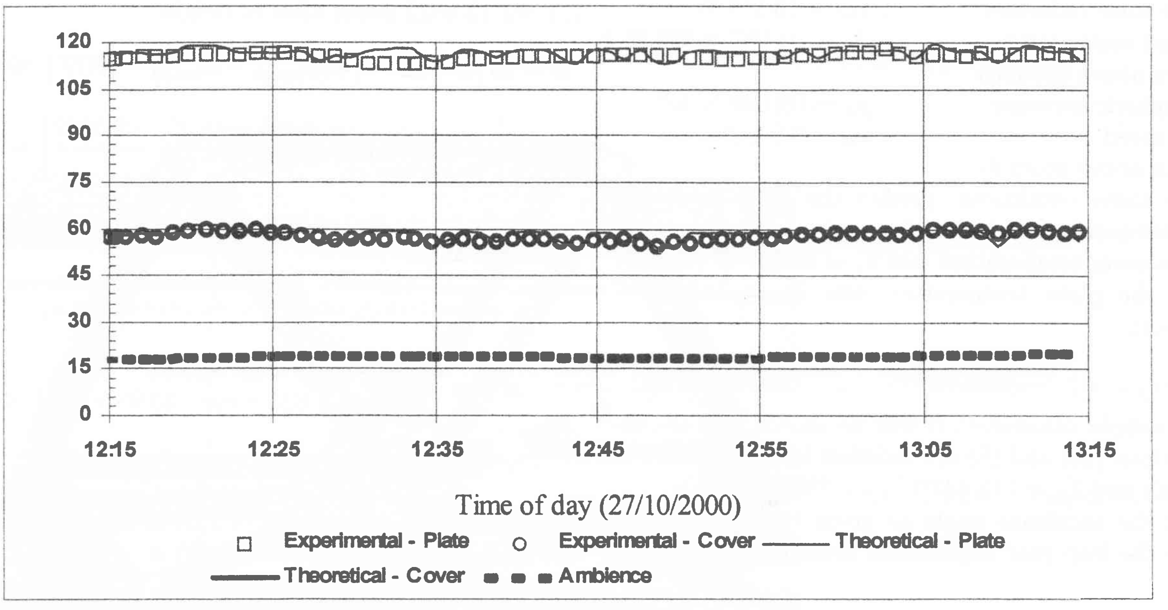


Fig. 3. Experimental and theoretical cover and plate temperatures of solar collector

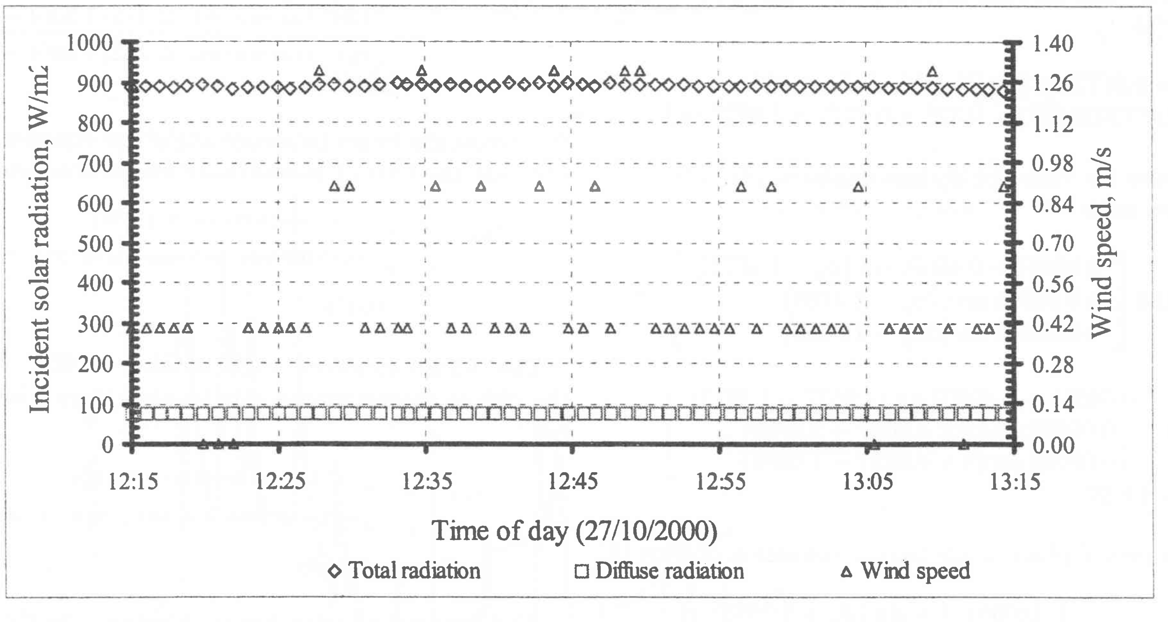


Fig. 4. Experimentally measured incident solar radiation and wind speed

Ambient conditions are as follows:

| | |
|---|---|
| Total incident solar radiation | $I_h = 895.47$ W/m$^2$ |
| Diffuse solar radiation | $I_{hd} = 76.115$ W/m$^2$ |
| Ambient temperature (1.05 m above ground) | $T_a = 19.1°C = 292.25$ K |
| Atmospheric pressure | $p_a = 101049$ N/m$^2$ |
| Wind speed (1.05 m above ground) | $v_w = 0.9$ m/s |

For the above conditions, predict the glass cover and the plate temperatures (during the particular test the measured glass cover temperature was $T_c = 56.537°C$ (329.687 K) while the plate temperature was $T_p = 115.448°C$ (388.598 K)).

SOLUTION

In this sample calculation it will be shown that the energy equations (13) and (8) are satisfied for $T_c = 56.690°C$ (329.840 K) and $T_p = 115.482°C$ (388.632 K).

To find the incidence angle as given by equation (3), determine the leap year adjustment according to equation (6).

$$YADJ = 0.25\left[2.5 - \{Y - 4\left(\text{integer}\left[(Y-1)/4\right]\right)\}\right]$$

$$= 0.25\left[2.5 - \{2000 - 4\left(\text{integer}\left[(2000-1)/4\right]\right)\}\right]$$

$$= -0.375 \text{ days}$$

with the integer function yielding a value of 499.

With this value find the annual phase angle according to equation (5).

$$\phi_p = 0.0172028\,(DOY + YADJ + 0.417)$$
$$= 0.0172028\,(271 - 0.375 + 0.417) = 4.6627 \text{ rad}$$

Substitute the value for $\phi_p$ into equation (4) to find the declination angle

$$\phi_d = 180\begin{bmatrix} 0.00661 + 0.40602\sin(\phi_p - 1.4075) \\ +0.00665\sin(2\phi_p - 1.4789) \\ +0.00298\sin(3\phi_p - 1.0996)\end{bmatrix}/\pi$$

$$= 180\begin{bmatrix} 0.00661 + 0.40602\sin(4.6627 - 1.4075) \\ +0.00665\sin(2\times 4.6627 - 1.4789) \\ +0.00298\sin(3\times 4.6627 - 1.0996)\end{bmatrix}/\pi = -1.823°$$

Substitute the phase angle into the equation of time (8)

$$EOT = 1440\begin{bmatrix} 0.005114 + \sin(\phi_p + 3.0593) \\ +0.006892\sin(2\phi_p + 3.4646) \\ +0.000222\sin(3\phi_p + 3.3858) \\ +0.000153\sin(4\phi_p + 3.7766)\end{bmatrix}/\pi$$

$$= 1440\begin{bmatrix} 0.005114 + \sin(4.6627 + 3.0593) \\ +0.006892\sin(2\times 4.6627 + 3.4646) \\ +0.000222\sin(3\times 4.6627 + 3.3858) \\ +0.000153\sin(4\times 4.6627 + 3.7766)\end{bmatrix} = 9.0922 \text{ min}$$

Using the value obtained for the equation of time value, the hour angle can be determined according to equation (7), for 12.6622 hours after midnight

$$\omega = 15\left[\psi - 12 - \{4\left(\phi_{longs} - \phi_{long}\right) - EOT\}/60\right]$$

$$= 15\left[12.6622 - 12 - \frac{4(30 - 18.85) - 9.0922}{60}\right] = 1.0561°$$

Substitute $\phi_d$ and $\omega$ into equation (3) to find the beam incidence angle

$$\theta = \arccos\left(\sin\phi_d \sin\phi_1 + \cos\phi_d \cos\phi_1 \cos\omega\right)$$

$$= \arccos\begin{pmatrix} \sin -1.823 \times \sin -33.98 \\ +\cos -1.823 \times \cos -33.98 \times \cos 1.0561\end{pmatrix}$$

$$= 32.172°$$

Therefore the beam reflection from the upper surface of the glass according to equation (2) is

$$\rho_b = \left[\frac{\sin^2\{\arcsin(\sin\theta/1.526) - \theta\}}{\sin^2\{\arcsin(\sin\theta/1.526 + \theta)\}}\right] + \frac{1}{2}\left[\frac{\tan^2\{\arcsin(\sin\theta/1.526) - \theta\}}{\tan^2\{\arcsin(\sin\theta/1.526) + \theta\}}\right]$$

$$\rho_b = \left[\frac{\sin^2\{\arcsin(\sin 32.172/1.526) - 32.172\}}{\sin^2\{\arcsin(\sin 32.172/1.526 + 32.172)\}}\right] + \frac{1}{2}\left[\frac{\tan^2\{\arcsin(\sin 32.172/1.526) - 32.172\}}{\tan^2\{\arcsin(\sin 32.172/1.526) + 32.172\}}\right] = 0.045514$$

Substitute the beam incidence angle into equation (9) and find that the beam transmittance due to absorptance

$$\tau_{ba} = e^{-C_e t_c/\cos\{\arcsin(\sin\theta/1.526)\}} = e^{-13\times 0.000388/\cos\{\arcsin(\sin 32.172/1.526)\}} = 0.9476$$

The diffuse incidence angle is taken as 60°. Therefore the diffuse transmittance due to absorptance is found to be

$$\tau_{ba} = e^{-C_e t_c/\cos\{\arcsin(\sin\theta/1.526)\}} = e^{-13\times 0.000388/\cos\{\arcsin(\sin 60/1.526)\}} = 0.9406$$

The fraction of solar beam radiation absorbed by the plate is according to equation (10)

$$(\tau_b\alpha_p)_b = (1-\rho_b)^2\tau_{ba}\alpha_p/\left[\frac{\{1-\rho_d(1-\alpha_p)\}}{(1-\rho_b^2\tau_{ba}^2)}\right]$$

$$= (1 - 0.045514)^2 \times 0.9476 \times 0.9/\left[\frac{\{1 - 0.09346\,(1-0.9)\}}{(1 - 0.045514^2 \times 0.9476^2)}\right]$$

$$= 0.78576$$

while the fraction of diffuse solar energy absorbed by the plate is according to equation (11)

$$\begin{aligned}&(\tau_d\alpha_p)_d = 0.8218\tau_{da}\alpha_p \\ &/\left[(0.90654+0.09346\alpha_p)\left(1-0.00873\tau_{da}^2\right)\right] \\ &= 0.8218\times 0.9406\times 0.9 \\ &/\left[(0.90654+0.09346\times 0.9)\left(1-0.00873\times 0.9406^2\right)\right] \\ &= 0.70771\end{aligned}$$

Using the predicted plate and cover temperatures, the thermophysical properties and the Rayleigh number of the entrapped air are evaluated at a mean cover-plate temperature $T_{pc} = (T_p+T_c)/2 = (388.632+329.84)/2 = 359.23$K.

At this temperature find the air density $\rho_{pc} = 0.9798$ kg/m$^3$, the specific heat capacity $c_{ppc} = 1010.3$ J/kg.K, the thermal conductivity $k_{pc} = 0.030684$ W/m.K and the dynamic viscosity $\mu_{pc} = 2.1111\times 10^{-5}$ kg/m.s.

The Rayleigh number from equation (16) is

$$\begin{aligned}Ra_{pc} &= 2\left(T_p-T_c\right)gt_a^3\rho_{pc}^2c_{ppc} \\ &\quad /\left[\left(T_p+T_c\right)\mu_{pc}k_{pc}\right] \\ &= 2\left(388.632-329.84\right)\times 9.81 \\ &\quad \times 0.0395^3\times 0.9798^2\times 1010.3 \\ &\quad /\left[\begin{array}{l}(388.632+329.84)\times 2.1111 \\ \times 10^{-5}\times 0.030684\end{array}\right] \\ &= 148.150\times 10^3\end{aligned}$$

Substitute this value for the Rayleigh number into equation (15) and find the heat transfer coefficient

$$\begin{aligned}h_{pc} &= \left[\begin{array}{l}1+1.44\left(1-1708/Ra_{pc}\right) \\ +\left\{\left(Ra_{pc}/5830\right)^{1/3}-1\right\}\end{array}\right]k_{pc}/t_a \\ &= \left[\begin{array}{l}1+1.44\left(1-1708/148.150\times 10^3\right) \\ +\left\{\left(148.150\times 10^3/5830\right)^{1/3}-1\right\}\end{array}\right] \\ &\quad \times 0.030677/0.0395 \\ &= 3.389\text{W/m}^2\text{K}\end{aligned}$$

From equation (12) it follows that the left-hand side of equation (13) yields a value of

$$\begin{aligned}q_p &= (\tau_b\alpha_p)_b I_{hb} + (\tau_d\alpha_p)_d I_{hd} \\ &= 0.78576\left(895.47-76.115\right)+0.70771\times 76.115 \\ &= 697.6 \text{ W/m}^2\end{aligned}$$

and the right-hand side of equation (13), according to equation (14) and (17), gives

$$\begin{aligned}q_{ph}+q_{pc} &= h_{pc}\left(T_p-T_c\right)+\sigma\left(T_p^4-T_c^4\right) \\ &\quad /\left(1/\varepsilon_p+1/\varepsilon_c+1\right) \\ &= 3.389\left(388.632-329.84\right) \\ &\quad +5.67\times 10^{-8}\left(388.632^4-329.84^4\right) \\ &\quad /\left(1/0.9+1/0.88+1\right) \\ &= 697.8 \text{ W/m}^2\end{aligned}$$

Hence, the plate energy equation for the solar collector is satisfied.

To evaluate the energy equation (18) applicable to the glass cover employ the predicted cover temperature $T_c$ to find the thermophysical properties and dimensionless Prandtl, Grashof and Reynolds numbers of the air, at a mean air-cover temperature $T_{ac} = (T_c+T_a)/2 = (329.84+292.25)/2 = 311.05$ K. At this temperature the density of the air is $\rho_{ca} = 1.13162$ kg/m$^3$, the specific heat $c_{pca} = 1007.1$ J/kg.K, the thermal conductivity $k_{ca} = 0.027069$ W/m.K , the dynamic viscosity $\mu_{ca} = 1.8974\times 10^{-5}$ kg/m.s and the Prandtl number is

$$\begin{aligned}\Pr_{ca} &= \frac{\mu_{ca}c_{pca}}{k_{ca}} = \frac{1.8974\times 10^{-5}\times 1007.1}{0.027069} \\ &= 0.70593\end{aligned}$$

The Grashof number is given by

$$\begin{aligned}Gr_{ca} &= \frac{2\left(T_c-T_a\right)gL_c^3\rho_{ca}^2}{\left(T_c+T_a\right)\mu_{ca}^2} \\ &= \frac{2\left(329.84-292.25\right)\times 9.8\times 1.0^3\times 1.13162^2}{\left(329.84+292.25\right)\times\left(1.8974\times 10^{-5}\right)^2} \\ &= 4.21698\times 10^9\end{aligned}$$

while the Reynolds number is

$$\begin{aligned}\text{Re}_{ca} &= \frac{\rho_{ca}v_wL_c}{\mu_{ca}} = \frac{1.13162\times 0.9\times 1.0}{1.8974\times 10^{-5}} \\ &= 53676.5\end{aligned}$$

Substitute the previously determined glass solar characteristics into the left-hand side of equation (18) and find

$$\frac{(1-\rho_b)(1-\tau_{ba})}{(1-\rho_b\tau_{ba})}I_{hb}+\frac{(1-\rho_d)(1-\tau_{da})}{(1-\rho_d\tau_{da})}I_{hd}+q_{ch}+q_{pc}$$

$$=\frac{(1-0.045514)(1-0.9476)}{(1-0.045514\times 0.9476)}(895.47-76.115)$$

$$+\frac{(1-0.09346)(1-0.9406)}{(1-0.09346\times 0.9406)}76.115$$

$$+3.389\left(388.632-329.84\right)$$

$$+5.67\times 10^{-8}\left(388.632^4-329.84^4\right)/\left(1/0.9+1/0.88-1\right)$$

$$= 743.9 \text{ W/m}^2$$

Upon substituting the above values for the Prandtl, Grashof and Reynolds numbers into equation (20), the right-hand side of equation (18) yields a value of

$$(q_{cs} + q_{ca}) = \varepsilon_c \sigma \left[T_c^4 - \left(0.0552 T_a^{1.5}\right)^4\right]$$

$$+ \left[\left(Gr_{ca}\,\mathrm{Pr}_{ca}\right)^{1/3}\left(0.227 + 1.406 \times 10^{-6} Ra_{ca}\right)\right]$$

$$\times k_{ca}\left(T_c - T_a\right)/L_c$$

$$= 0.88 \times 5.67 \times 10^{-8} \left[\begin{array}{l} 329.84^4 \\ -\left(0.0552 \times 292.25^{1.5}\right)^4 \end{array}\right]$$

$$+ \left[\begin{array}{l} \left(4.21698 \times 10^9 \times 0.7059\right)^{1/3} \\ \times\left(0.227 + 1.406 \times 10^{-6} \times 53676.5\right) \end{array}\right]$$

$$\times 0.02707 \times (329.84 - 292.25)/1.0$$

$$= 743.9 \text{ W/m}^2$$

As in the case of the solar collector plate, it is shown that the predicted temperatures satisfy the cover energy equation for the simple collector.