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Thermodynamic study of a simplified model of the solar chimney power plant

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Abstract

A simplified model of solar chimney power plant (SCPP) consists of a heating air collector, turbine and chimney. Thermodynamic interpretation of processes occurring in these SCPP components is based on the derived energy and exergy balances. The examples of the energy and exergy flow diagrams show how the SCPP input of 36.81 MW energy of solar radiation, corresponding to 32.41 MW input of radiation exergy, is distributed between the SCPP components. Responsive trends to the varying input parameters are studied. Additionally, the concept of mechanical exergy (ezergy) of air is applied and it allowed for quantitative determination of the effect attributed to the terrestrial gravity field on the component processes of the SCPP.

Keywords: Engineering thermodynamics; Exergy; Solar chimney power plant

1. Introduction

A typical solar chimney power plant (SCPP) consists of a circular greenhouse type collector and a tall chimney at its centre. Air flowing radially inwards under the collector deck is heated from the collector floor and deck, and through a turbine enters the chimney.

Fig. 1 describes example of SCPP taken for consideration in the present study. A draft-driven environmental air (point 0) enters the collector through the gap of height H_e. The collector floor of diameter D_f is under the transparent deck which declines appropriately to ensure the constant radial cross-section area for the radially directed flow of the air. The assumption of constant cross-section area in the collector means that $\pi \times D_f \times H_e = \pi \times D_1 \times H_1 = \pi \times D_1^{2/4}$, and so, the assumed value H_e allows for calculation of the inlet turbine diameter $D_1 = (4 \times H_e \times D_f)^{1/2}$ and height $H_1 = D_1/4$. Collector floor preheats air from state 0 to state 1 (state 1 prevails in the zone denoted with dashed line). Preheated air (state 1) expands in turbine to state 2. The turbine inlet and outlet diameters are D_1 and D_2, respectively. The height of turbine is H_T; ($H_1 + H_T = H_2$). Expanded air leaves the SCPP (at point 3) through the chimney at height H_3.

For the established geometrical parameters of the collector-turbine-chimney system, and for the constant thermodynamic input data, like solar radiation intensity and environment parameters, the system spontaneously self-models in response to actual situation. It means that buoyancy effect determines the flow rate of air through the system and all the air parameters; temperature and pressure along the air flow path.

Exact thermodynamic considerations of the SCPP require involving of many different theoretical areas like, e.g., thermodynamics, heat transfer and fluid mechanics. Especially the exergy analysis requires, e.g., taking into account temperature distribution over surfaces (floor, deck or chimney), require complex expression on exergy of non-equilibrium radiation occurring from these surfaces; each

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Nomenclature

A	surface area, m^2
a	9.7807 m/s^2: constant of Eq. (12)
B	exergy, W
b	exergy, %
b	-3.086×10^{-6} s^{-1}: constant of Eq. (12)
c_D	chimney wall thickness coefficient
c_p	specific heat at constant pressure, J/kg K
D	diameter, m
d	-9.973×10^{-5} kg/m^4: constant of Eq. (12)
E	energy, W
E	energy, %
e	=1.217 kg/m^3: constant of Eq. (12)
G	gravity input, W
g	gravitational acceleration, m/s^2
H	height or altitude, m
H_e	height of air inlet, m
H_T	height of turbine, m
h	convective heat transfer coefficient, W/m^2 K
i	successive number of row in Table 1
j	successive number of column in Table 1
k	thermal conductivity W/m K
M	output
m	air mass flow rate, kg/s
N	input
Nu	Nusselt number
P	power, W
Pr	Prandtl number
p	absolute pressure, Pa
R	287.04 J/kg K, individual gas constant (for air)
Re	Reynolds number
r	radial coordinate
r_T	relative pressure drop in turbine
S	solar radiosity, W/m^2
SCPP	solar chimney power plant
T	absolute temperature, K
w	flow velocity, m/s
Z	mechanical exergy (ezergy), W
z	mechanical exergy (ezergy), %

Greek symbols

α	absorptivity
β	angle, deg
ΔB	exergy loss, W
Δb	exergy loss, %
ε	emissivity
ϕ	radiation shape factor
η_T	internal efficiency of turbine
κ	isentropic exponent
ν	kinematic viscosity coefficient, m^2/s
ρ	density, kg/m^3
σ	5.6693×10^{-8} $W\,m^{-2}\,K^{-4}$: black body radiation constant
τ	transmissivity

Subscripts

a	air
ch	chimney
cv	convective
d	deck
E	effective
f	floor
G	gravity input
H	altitude
M	output
N	input
P	turbine power
p	potential
R	reflected
Q	heat
S	solar
Sky	sky
T	turbine
w	velocity (kinetic)
x, y	different surface
0	environment
1, 2, 3	localities shown in Fig. 1

element of the surface has different temperature and radiates different exergy. Energy analysis is a little simpler because energy of radiation of surface is calculated with simpler formula and the energy fluxes must not be followed separately for each particular temperature during successive reflections between grey surfaces (energy of radiation, in contrary to the exergy, does not distinguish radiation fluxes of different temperature). Complex, irregular (selective) spectra of radiating bodies (air, floor, partly transparent deck, etc.), make consideration additionally difficult. There are the problems of locally considered shape factor for radiation transfer, varying conditions for convective coefficients (e.g., dependence of materials properties on thermodynamic parameters; specific heat of air, viscosity coefficient, etc), – distribution of air flow velocity, in at least two-dimensional space, including possible turbulent behavior of air, – conservation of momentum and energy conversion from kinetic to potential forms. The processes can occur on unsteady mode especially when starting, restaring or shutting down periods. The boundary conditions and the environment conditions can vary significantly. All such factors are piling the difficulties of calculations.

Generally, there are three methods to obtain solution in exact study on thermodynamic objects. First; the analytical method, expected to produce solution of differential equations in analytical form (successful in very simplified case). From solving of differential problem the ordinary solvable equations can be obtained. Usually introduction of many

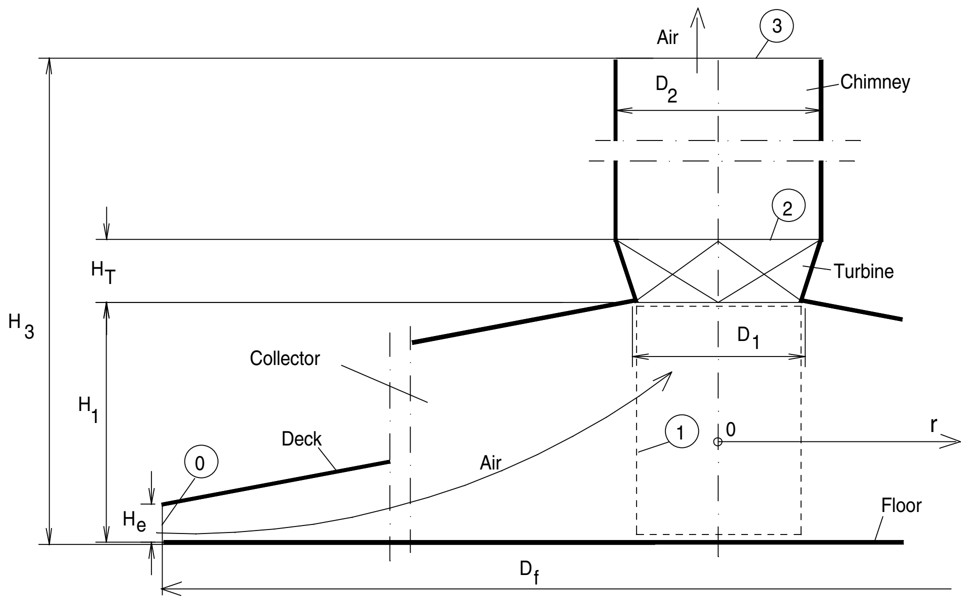

Fig. 1. Scheme of the considered SCPP.

simplifying assumptions allows for passing over the stage of formulation of differential equations and directly developing regular algebraic equations which can be solved if the number of unknowns is not larger than the number of derived equations. The present study belongs to this category.

Second; numerical method, solving numerically by developing differential equations into the finite difference equations, which allow for significantly less simplifying assumptions. However, the numerical method, replacing the analytical approach, itself brings some inadequacy by variables presented discretely.

Third; method based on similarity theory. According to this theory the characteristic dimensionless simplexes (similarity criteria) are extracted from differential equations. The criteria are used for derivation of mutual relations based on experimental data from the appropriately programmed measurements. The relations are fragmentary particular solutions, and have a meaning of particular integral of differential problem. In order to formulate an interpretative model of a process the similarity theory may apply experimental data obtained on laboratory, pilot or commercial scale. The method has not been applied yet to a SCPP.

SCPP belongs to the thermodynamic objects for which the fully developed application of any of these three methods is practically very difficult, especially when exergy is involved. Therefore, for engineering purposes the certain simplified models of processes is usually formulated. The models ignore some less important component phenomena, giving better chance for an achievable solution. The purpose of even simplified solution is to ensure possibility of measurable estimation of basic relations between process parameters, possibility of designing and selection of optimal versions, all without uncertain guesses.

Obtaining a complete and exact mathematical description of the SCPP from thermodynamic viewpoint is very complex. Formulation of the description based on the mathematical analysis leads to differential equations for mass, momentum, energy conservation and exergy balance which, the more details of the processes are included into analysis, the more difficult are to solve. However, the more details considered, the higher is the significance of obtained solution. Therefore the optimization problem rises for selection of the analysis precision degree.

Often simplified models have to be applied based on assumption eliminating many of real features of the thermodynamic problem. Some models simplify more some less. For each model the expectation for exact results are diminishing with growing numbers of assumptions. However the simplified models usually describe at least the trends and its intensity of response of dependent object parameters to the varying input parameters of the object. Often such achievement is quite satisfactory for practical purposes.

The idea of SCPP was initiated by Schlaich in the late 1970s. The first 36 kW pilot SCPP plant began operation in Manzanares, near Madrid, Spain. The chimney was 195 m high and the radius of the collector was 120 m. Gradually the SCPP concept was developed and many publications discussed various aspects of solar plant, in which complex processes of heat transfer and fluid mechanics occur. For example, Pasumarthi and Sherif (1998a,b) developed a mathematical model of the collector for studying the temperature and flow velocity of heated air and determining the expected SCPP efficiency and power. Padki and Sherif (1992) considered the effects of geometrical parameters of the chimney performance. The temperature and pressure fields for flowing air were studied analytically, e.g., by Ming et al. (2006), or analytically and numerically

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by Pastohr et al. (2004), with estimating also the varying temperature of ground and heat transfer coefficients. Many studies are also focused on the pressure drop across the turbine as a part of the total available pressure difference in the system. The pressure drop was evaluated, e.g., by Haaf et al. (1983), Mullet (1987), Schlaich (1995), Gannon and Von Backström (2000), Bernardes et al. (2003), and recently by Von Backström and Fluri (2006). Thermodynamic variables, regarding chimney, dependent on wall friction, additional losses, internal drag, cross-section area for chimneys as tall as 1500 m were studied by Von Backström and Gannon (2000). Exergy interpretation of chimney process was analyzed by Petela (2008d). However, it is noteworthy that, due to complexity of the object, all existing publications consider certain selected aspects of SCPP but neither of these publications considers the entire SCPP with all its components.

The present work is an attempt to extend the existing literature on SCPP by thermodynamic analysis of energy distribution in the whole SCPP and also by including, for the first time, exergetic approach. Fundamentals of classic exergy analysis are presented, e.g., by Szargut and Petela (1965, 1968) or Szargut et al. (1988). Significance of thermodynamic analysis for various processes is well discussed recently by Petela (2008a).

The objective of the presented study is to outline a simplified interpretative mathematical model of the SCPP which could be used to demonstrate feasibility of application of exergy for analysis of SCPP and for proposing the methodology of the full thermodynamic analysis including exergy. The study presents also application of the concepts of ezergy and gravity input for the modified exergetic interpretation of processes.

2. The main assumptions for the simplified mathematical model of the SCPP

Generally, there is no detailed, ready for using, consistent data on the all SCPP parameters. Therefore the required data are generated by the proposed simplified model which is considered under the following main assumptions:

(i) Floor has no heat loss to the soil; is perfectly insulated, and is perfectly black (emissivity $\varepsilon_f = 1$). Thus, there is no solar energy reflected from the floor. It is noteworthy to mention that the further simplification, not applied in the present consideration, could be assumption of the floor material of almost infinitely large conductivity which then could motivate the assumption of the constant temperature of the floor in the whole collector.

(ii) Deck material is prepared in such way that it is almost perfectly transparent for solar radiation (transmissivity $\tau_d = 0.95$) and the remaining part (5%) of solar radiation arriving at the deck is reflected. However, the deck material absorbs perfectly (absorptivity $\alpha = 1$) any low temperature radiation, e.g., from the floor. Thus, consideration of multi-reflected radiation fluxes is simplified. In addition, the deck is enough thin that heat conducted through the deck occurs at zero temperature gradient. The properties of deck were assumed for better exposing the effect of trapping solar radiation energy within the collector.

(iii) Variation of the floor temperature with the radial coordinate r was analyzed by Pastohr et al. (2004). Dependent of the used model they received relatively significantly different distributions of the temperature, not clearly explained. The literature data on temperature distribution at the deck surface is very limited. Therefore, for the present study, certain "effective temperature" T_E of floor or deck is applied with the following definition equation, which includes potentials to the heat transfer by conduction and radiation:

$$A(h_{\mathrm{average}} T_E + \sigma T_E^4) = \int_A h_{\mathrm{local}} T \, dA + \sigma \int_A T^4 \, dA \qquad (1)$$

where h_{average} and h_{local} are the average and local convective heat transfer coefficients, respectively, σ is the Stefan–Boltzmann constant, A is the surface area and T is the local surface temperature. As some computation estimations indicate, the concept of effective temperature can be applied only when the surface temperature varies within not too large range, just expectedly like in case of the considered floor and deck.

(iv) Chimney material is perfectly black. The chimney wall is thin thus there is no temperature gradient along the wall thickness and both sides of chimney (inner and outer) has the same temperature constant along the chimney height.

(v) Distribution of air temperature is represented by certain effective temperature defined according to Eq. (1), however with excluded radiative heat transfer.

(vi) Air is considered as an ideal gas of which parameters fulfill the state equation; $p = \rho \times R \times T$, and the specific heat is assumed constant, (not varying with temperature).

(vii) Air is almost perfectly transparent for radiation, (transmissivity $\tau_a \approx 1$ and emissivity $\varepsilon_a \approx 0$). Air can exchange heat only by convection and conduction.

(viii) Air flow in the whole SCPP is frictionless. The relative air pressure drop r_T during expansion in turbine is differently estimated by many authors, as discussed by Von Backström and Fluri (2006). The drop is considered in the range from 0.66, e.g., by Mullet (1987), to 0.97 by Bernardes et al. (2003). According to investigation by Von Backström and Fluri (2006), "for maximum fluid power, the optimum ratio" $r_T = 2/3$. Therefore in the present consideration it is assumed:

$$\frac{p_1 - p_2}{p_1 - p_3} = r_T = \frac{2}{3} \qquad (2)$$

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Optimization of the SCPP is not the objective of the present work and only suggested optimum value r_T is assumed for analysis of the SCPP. For illustration purpose, based on later calculation results (Table 1a, column 4), the distribution of environment pressure and the pressure of air along its flow within the SCPP is shown in Fig. 2. The air pressure of environment (solid thin line) drops from p_0, at the zero level, to p_3, at the level H_3 of chimney inlet. The air pressure inside the SCPP drops from p_0 to p_1 at the collector outlet (dashed line) and it is assumed that the same pressure p_1 prevails also at the inlet to the turbine. Then, within turbine, air pressure (thick solid line) drops from p_1 to p_2 during adiabatic expansion generating power. Air from turbine flows upward and its pressure (another dashed line) achieves value p_3 at the chimney exit.

(ix) Using average values of gravitational acceleration and air density along the height H_3:

$$p_3 = p_0 - \frac{g_0 + g_3}{2}\,\frac{\rho_0 + \rho_{03}}{2} H_3 \tag{3}$$

where the following approximations, used by Petela (2008b), were applied: $g_{H3} = g_0 - 3.086 \times 10^{-6} \times H_3$ and $\rho_{H3} = \rho_0 - 9.973 \times 10^{-5} \times H_3$. At the earth surface the atmospheric pressure $p_0 = 101.235$ kPa and gravity acceleration $g_0 = 9.81$ m/s^2.

(x) Momentum conservation equation for the air flow within collector is derived as

$$p_0 - p_1 = \rho_{a1} w_1^2 \tag{4}$$

where ρ_{a1} and w_1 are the density and flow velocity of air at point 1.

(xi) Deck and chimney radiate to the space of sky temperature T_{sky}. Therefore, for a clear sky the Swinbank formula is applied: $T_{sky} = 0.0552 \times T_0^{1.5}$.

(xii) In order to obtain the fair comparison basis the reference state for calculation of energy are the same like for exergy: environment temperature $T_0 = 288.14$ K (15 °C), and environment pressure $p_0 = 101.235$ kPa.

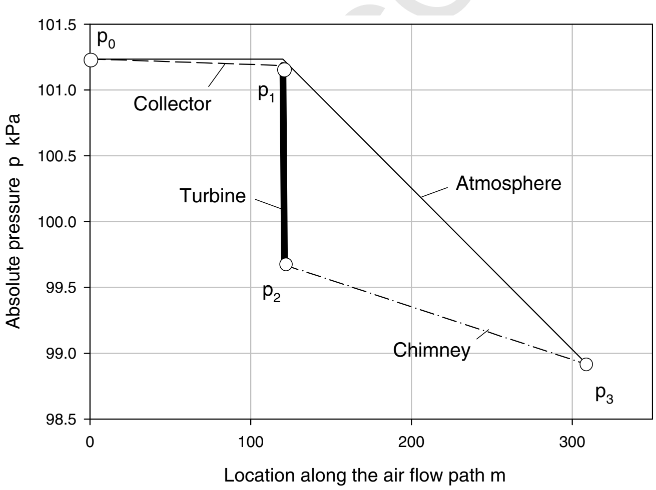

Fig. 2. Distribution of the absolute pressure in the considered SCPP.

More assumptions are discussed in the following chapters.

3. Energy analysis

Energy analysis is based on the energy conservation equations. The energies E are used in six equations written successively for: floor surface, air in collector, collector (including floor, air and deck), turbine, chimney and chimney surface:

$$E_{S\text{-}f} = E_{f\text{-}a} + E_{f\text{-}d} \tag{5}$$

$$E_{f\text{-}a} + E_{d\text{-}a} = E_{a1} + E_{w1} + E_{p1} \tag{6}$$

$$E_{S\text{-}f} = E_{a1} + E_{w1} + E_{p1} + E_{d\text{-}sky} + E_{d\text{-}0} + E_{d\text{-}ch} \tag{7}$$

$$E_{a1} + E_{w1} + E_{p1} = E_{a2} + E_{w2} + E_{p2} + E_P \tag{8}$$

$$E_{a2} + E_{w2} + E_{p2} + E_{d\text{-}ch} = E_{a3} + E_{w3} + E_{p3} + E_{ch\text{-}0} + E_{ch\text{-}sky} + E_{ch\text{-}gr} \tag{9}$$

$$E_{a\text{-}ch} + E_{d\text{-}ch} = E_{ch\text{-}0} + E_{d\text{-}sky} + E_{ch\text{-}gr} \tag{10}$$

Energies E have the following subscripts:

S-f solar radiation arriving at the floor,
f-a convection heat from floor to air,
f-d energy exchanged by radiation between floor and deck,
d-a convection heat from deck to air,
d-sky energy exchanged by radiation between deck and sky
d-0 convection heat from deck to atmosphere,
d-ch energy exchanged by radiation between deck and chimney,
ch-0 convection heat from chimney surface to atmosphere,
ch-sky energy exchanged by radiation between chimney surface and sky,
ch-gr energy exchanged by radiation between chimney surface and ground
a-ch heat transferred from chimney air to the chimney surface,
1a, 2a, 3a enthalpy of air at point 1, 2 and 3,
w1, w2, w3 kinetic energy due to the air flow velocity w1, w2 and w3,
p1, p2, p3 potential energy of air at point 1, 2 and 3,
P turbine power

Kinetic energies are calculated as $E_w = m \times w^2/2$, where m is the air mass flow rate; $m = 0.25 \times \pi \times D_1^2 \times w_1 \times \rho_{a1}$. Enthalpy of air is $E_a = m \times c_p \times (T_a - T_0)$ where c_p is the specific heat of air at constant pressure.

Potential energy of considered air, at its constant density ρ, depends on the altitudinal variation of atmospheric air density and gravity acceleration. The solution of differential formula on potential energy E_p, J/kg, was derived, e.g., by Petela (2008a,b,c,d):

$$E_p = m\left\{-\frac{1}{\rho d}\left[\frac{b}{6d}(\rho - e)^3 + \frac{a}{2}(\rho - e)^2\right]\right\} \tag{11}$$

where a, b, e and d are constant values (see Nomenclature).

Total solar energy received by the floor is

$$E_{\text{S-f}} = \tau_d \varepsilon_f S A_d \tag{12}$$

where S, W/m^2, is the solar radiosity at the earth surface, τ_d is the transmissivity of deck, and ε_f is the floor emissivity ($\varepsilon_f = 1$).

Energy exchanged by radiation between deck and chimney:

$$E_{\text{d-ch}} = \varepsilon_d \varphi_{\text{d-ch}} \frac{\pi}{4}[D_f^2 - (c_D D_2)^2]\sigma(T_{dE}^4 - T_{ch}^4) \tag{13}$$

where $\sigma = 5.667 \times 10^{-8}$ W m^{-2} K^{-4} is the Stefan–Boltzmann constant, T_{dE} is the effective temperature of the deck, and c_D is the factor to account on thickness of the chimney wall. The radiation shape factor $\phi_{\text{d-ch}}$ can be calculated from reciprocity relation:

$$\varphi_{\text{d-ch}} \frac{\pi}{4}[D_f^2 - (c_D D_2)^2] = \varphi_{\text{ch-d}} \pi c_D D_2 (H_3 - H_2) \tag{14}$$

It can be derived that $\phi_{\text{ch-d}} = 0.5 \times (90 - \beta)/90$ where the angle β is determined by $\tan\beta = 2 \times H_3/D_f$.

Energy exchanged by radiation between floor and deck:

$$E_{\text{f-d}} = A_d \sigma (T_{fE}^4 - T_{dE}^4) \tag{15}$$

where T_{fE} is the effective temperature of the floor and surface area $A_d = \pi \times (D_f^2 - D_1^2)/4$.

The following formulae are applied for convection heat transfer fromFloor to air:

$$E_{\text{f-a}} = A_d h_{\text{f-a}} (T_{fE} - T_{aE}) \tag{16}$$

Deck to air:

$$E_{\text{d-a}} = A_d h_{\text{d-a}} (T_{dE} - T_{aE}) \tag{17}$$

Deck to environment:

$$E_{\text{d-0}} = A_d h_{\text{d-0}} (T_{dE} - T_0) \tag{18}$$

Chimney to environment:

$$E_{\text{ch-0}} = A_{ch} h_{\text{ch-0}} (T_{ch} - T_0) \tag{19}$$

and Chimney air to chimney wall:

$$E_{\text{a-ch}} = \pi D_2 (H_3 - H_2) h_{\text{a-ch}} \left(\frac{T_{a2} + T_{a3}}{2} - T_{ch}\right) \tag{20}$$

where h is the respective coefficient and the chimney surface $A_{ch} = \pi \times c_D \times D_2 \times (H_3 - H_2)$. The coefficient $h_{\text{a-ch}}$ is determined as $h_{\text{a-ch}} = Nu \times k/D_2$ where $k = 0.0267$ W/m K is thermal conductivity of air and the Nusselt number $Nu = 0.023 \times Re^{0.8} \times Pr^{0.4}$ and where the Prandtl number for air is $Pr = 0.7$ and the Reynolds number $Re = w_2 D_2/v$ (kinematic viscosity coefficient for air $v = 1.6 \times 10^{-5}$ m^2/s).

In similar way is determined coefficient $h_{\text{f-a}}$. Although the air flow is driven by buoyancy effect, the forced convection mechanism of the air flow is assumed. Thus the calculations are based on the Reynolds number instead of the Grashof number. In calculations the average flow velocity of air is assumed. The effective diameter D_E for the air flow was assumed as the average ratio of the respective flow cross-section areas (A_1) multiplied by four, to the respective perimeter lengths L_0 or L_1; $D_E = 2 \times A_1/(1/L_0 + 1/L_1)$. It was assumed that $h_{\text{a-d}} = h_{\text{f-a}}$.

The following formulae are applied for energy exchange by radiation betweenFloor and deck:

$$E_{\text{f-d}} = A_d \sigma (T_{fE}^4 - T_{dE}^4) \tag{21}$$

Deck and chimney:

$$E_{\text{d-ch}} = \varphi_{\text{d-ch}} A_d \sigma (T_{dE}^4 - T_{ch}^4) \tag{22}$$

Deck and sky:

$$E_{\text{d-sky}} = \varphi_{\text{d-sky}} A_d \sigma (T_{dE}^4 - T_{sky}^4) \tag{23}$$

Chimney and sky:

$$E_{\text{ch-sky}} = \varphi_{\text{ch-sky}} A_{ch} \sigma (T_{ch}^4 - T_{sky}^4) \tag{24}$$

Chimney and ground beyond the floor:

$$E_{\text{ch-gr}} = \varphi_{\text{ch-gr}} A_{ch} \sigma (T_{ch}^4 - T_{gr}^4) \tag{25}$$

where the radiation shape factors fulfill the following relations:

$$\varphi_{\text{d-sky}} + \varphi_{\text{d-ch}} = 1 \tag{26}$$

$$\varphi_{\text{ch-sky}} + \varphi_{\text{ch-d}} + \varphi_{\text{ch-gr}} = 1 \tag{27}$$

The factors $\phi_{\text{ch-d}}$ and $\phi_{\text{d-ch}}$ are determined based on Eq. (14), whereas the configuration of chimney relative to sky determines factor $\phi_{\text{ch-sky}} = 0.5$.

Calculation of temperature T_{a2} is based on the equation for the isentropic expansion in turbine at assumed isentropic exponent κ for air and the internal efficiency of turbine η_T. Conversion of energy of air into electric power occurs at an overall efficiency η_o which would include additionally mechanical and electric efficiencies of the turbine-generator unit.

The energy calculations were carried out with additional assumptions. Air temperature distribution in collector is assumed linear and thus $T_{aE} = (T_0 + T_{a1})/2$. The diameter ratio $D_1/D_2 = 0.95$. Based on additional calculations the air temperature drop in the chimney can be estimated as proportional to the chimney surface and inversely proportional to the air mass rate: $T_{a2} - T_{a3} = 0.154 \times D_2 \times H_3/m$.

In example of computation with use of the presented model the following date is assumed. Addressing the consideration to the pilot plant at Manzanares the floor diameter is $D_f = 240$ m and the chimney height $H_3 = 195$ m. Other data is as follows:

$$S = 800\ \text{W/m}^2,\ \ T_{gr} = T_0,\ \ c_D = 1.015,\ \ c_p = 1000\ \text{J/kg K}$$
$$\kappa = 1.4,\ \ \eta_T = 0.7,\ \ R = 287.04\ \text{J/kg K},\ \ H_T = 1\ \text{m}$$
$$h_{\text{ch-0}} = 7\ \text{W/m}^2\ \text{K},\ \ h_{\text{d-0}} = 5\ \text{W/m}^2\ \text{K},\ \ H_e = 0.3\ \text{m}$$

Table 1a
Responsive trends of some output parameters to change of some input parameters (Study I)

#	Quantity	Units	Ref. value	Mono-variant changes of input parameters and resulting output			
1	2	3	4	5	6	7	8
2	*Input*						
3	*S*	W/m^2	**800**	**850**	800	800	800
4	H_3	m	**195**	195	**200**	195	195
5	D_f	m	**240**	240	240	**250**	240
6	*He*	m	**0.3**	0.3	0.3	0.3	**0.35**
7	*Output*						
8	p1	Pa	101,233.66	101,233.93	101,233.67	101,233.83	10,1231.82
9	p2	Pa	99,686.13	99,686.22	99,646.76	99,686.19	99,685.52
10	p3	Pa	98,912	98,912	98,853	98,912	98,912
11	w1	m/s	1.10	0.99	1.09	1.03	1.67
12	m	kg/s	276	245	274	268	501
13	TfE	K	388.3	394.5	388.4	389.3	381.7
14	TdE	K	329.8	333.8	329.9	330.6	325.0
15	TaE	K	303.18	304.62	303.20	304.02	299.20
16	Ta1	K	318.19	321.08	318.24	319.89	310.25
17	Tch	K	292.43	292.90	292.35	292.65	291.81
18	*Energy*						
19	ea1	%	22.99	21.09	22.92	21.75	30.78
20	ea2	%	22.24	20.45	22.15	21.07	29.45
21	ea3	%	20.75	19.05	20.63	19.67	27.84
22	ew1	%	4.63e−4	3.11e−4	4.56e−4	3.64e−4	1.95e−3
23	ew2	%	1.93e−4	1.30e−4	1.91e−4	1.52e−4	8.12e−4
24	ew3	%	3.88e−4	2.61e−4	3.83e−4	0.0003.05e−4	1.64e−3
25	ep1	%	0.3994	0. 4020	0.3988	0.3995	0.3871
26	ep2	%	0.5118	0.5046	0.5139	0.5055	0.5396
27	ep3	%	0.5278	0.5129	0.5300	0.5180	0.5951
28	efa	%	17.81	16.23	17.76	16.89	23.73
29	efd	%	77.19	78.77	77.24	78.11	71.27
30	eda	%	5.577	5.259	5.562	5.258	7.435
31	ed0	%	26.05	26.82	26.07	26.52	23.05
32	edsky	%	44.21	45.27	44.24	45.00	39.51
33	edch	%	1.35	1.42	1.36	1.34	1.26
34	ech0	%	0.90	0.94	0.90	0.89	0.83
35	echsky	%	1.70	1.64	1.74	1.61	1.78
36	echgr	%	0.230	0.241	0.235	0.224	0.212
37	EP	kW	229	203	234	222	423
38	eP (η_e)	%	0.64	0.53	0.65	0.57	1.18
39	*Exergy*						
40	ba1	%	1.2443	1.2439	1.2423	1.2393	1.2434
41	ba2	%	0.08	0.27	0.06	0.20	−0.83
42	ba3	%	−0.61	−0.34	−0.65	−0.44	−1.95
43	bw1	%	5.148e−4	3.46e−4	5.07e−4	4.05e−4	2.16e−3
44	bw2	%	2.15e−4	1.44e−4	2.12e−4	1.69e−4	9.03e−4
45	bw3	%	4.31e−6	2.89e−6	4.25e−6	3.39e−6	1.82e−5
46	bp1	%	0.4438	0.4467	0.4431	0.4439	0.4301
47	bp2	%	0.5687	0.5607	0.5710	0.5617	0.5995
48	bp3	%	0.00586	0.00570	0.005889	0.00576	0.00661
49	bfa	%	5.10	4.86	5.09	4.88	6.46
50	bfd	%	17.24	18.58	17.26	17.62	14.89
51	bda	%	0.783	0.798	0.782	0.750	0.937
52	bd0	%	3.657	4.072	3.664	3.781	2.907
53	bdsky	%	2.237	2.640	2.242	2.343	1.623
54	bdch	%	0.114	0.131	0.115	0.115	0.095
55	bch0	%	0.0145	0.0168	0.0144	0.0151	0.0115
56	bchsky	%	−0.044	−0.041	−0.046	−0.042	−0.049
57	bchgr	%	0.182	0.215	0.189	0.176	0.143
58	Δbf	%	72.66	76.55	77.65	77.51	78.65
59	Δba	%	4.20	3.97	4.19	3.94	5.73
60	Δbd	%	10.4496	10.9423	10.4600	10.6287	9.3263
61	ΔbT	%	0.33	0.28	0.34	0.29	0.60
62	Δbch	%	1.21	1.10	1.23	1.16	1.70
63	bP (η_b)	%	0.70	0.59	0.73	0.63	1.31

(*continued on next page*)

Table 1a (*continued*)

#	Quantity	Units	Ref. value	Mono-variant changes of input parameters and resulting output			
64	*Ezergy*						
65	za1	%	8.96	8.37	8.94	8.57	11.42
66	za2	%	8.82	8.25	8.79	8.44	11.11
67	za3	%	8.24	7.68	8.19	7.88	10.56
68	zGa	%	7.28	6.68	7.26	6.89	9.75
69	zGT	%	0.89	0.75	0.91	0.80	1.60
70	zGch	%	0.673	0.593	0.675	0.632	1.160

The model responses to the input parameters as presented in Tables 1a and 1b. Column 4 of the tables present the basic input case (reference case) of which results are compared to the results of other input cases represented by columns 5–10 and discussed in Section 6. Column 4, with the relatively low power, presents the closest case to the power reported for the Manzanares pilot plant. The energy results of this column are used for the flow energy diagram (Fig. 3). The diagram shows of how the solar radiation energy arriving at the deck $E_S = 39.05$ MW, reduced by 5% reflection, is distributed between five SCPP components; collector air, floor, deck, turbine and chimney.

In the diagram the energy streams E, W, are represented by their percentage values e related to the solar radiation energy E_S. The floor (black body) fully absorbs the solar radiation (95.00%) transmitted through the deck and converts this radiation energy to the energy at the level of temperature T_{fE}. Part of this energy ($e_{f\text{-}d} = 77.19\%$) radiates to the deck and the rest $e_{f\text{-}a} = 17.81\%$ is transferred by convection to heated air in the collector. The power performed by turbine is relatively small ($E_P = 0.23$ MW) mostly due to the small mass flow rate of air ($m = 276$ kg/s) and due to small pressure drop during the air expansion. Percentage power of turbine $e_P = 0.64\%$ represents the energy efficiency η_e of the SCPP. The exhausted energy (enthalpy) of air from chimney is $e_{a3} = 20.75\%$ whereas the exhausted potential and kinetic energies are small; $e_{p3} = 0.52\%$ and $e_{w3} = 3.87 \times 10^{-4}\%$, respectively. The other SCPP energy losses are by radiation and convection heat transferred from deck and chimney to the sky and environment. Solar energy reflected from the deck is assumed $e_R = 5.00\%$.

4. Exergy analysis

Data obtained from energy analysis can be used for designing, operation and evaluation of the SCPP. However, the same processes, with the same data, can be also interpreted based on exergy analysis with use of exergy balance equations. Exergy B in these equations has the subscripts respectively to E in energy analysis. The five separate exergy equations can be written for floor, deck, air in collector, turbine and chimney. The exergy equations are analogical to energy equations and differ by the additional members, ΔB, representing the respective irreversible exergy losses:

$$B_{S\text{-}f} = B_{f\text{-}a} + B_{f\text{-}d} + \Delta B_f \quad (28)$$

$$B_{f\text{-}d} = B_{d\text{-}a} + B_{d\text{-}sky} + B_{d\text{-}0} + B_{d\text{-}ch} + \Delta B_d \quad (29)$$

$$B_{f\text{-}a} + B_{d\text{-}a} = B_{a1} + B_{w1} + B_{p1} + \Delta B_a \quad (30)$$

$$B_{a1} + B_{w1} + B_{p1} = B_{a2} + B_{w2} + B_{p2} + B_P + \Delta B_T \quad (31)$$

$$B_{a2} + B_{w2} + B_{p2} + B_{d\text{-}ch} = B_{a3} + B_{w3} + B_{p3} + B_{ch\text{-}0} + B_{ch\text{-}sky} + B_{ch\text{-}gr} + \Delta B_{ch} \quad (32)$$

Exergy of solar radiation can be estimated for the radiation temperature slightly smaller than 6000 K. According to data by Petela (1964) the exergy B_S could be assumed as about 90% of radiation energy; $B_S = 0.9 \times E_S$.

Generally, radiation exergy B of a surface at its temperature T, emissivity ε and surface area A, is determined according to Petela (1964), or Petela (2003):

$$B = \varphi A \varepsilon \frac{\sigma}{3}(3T^4 + T_0^4 - T_0 T^3) \quad (33)$$

where ϕ is the radiation shape factor accounting for geometrical configuration of the considered surface regarding to an eventual surface at which considered radiation would arrive. Exergy of radiation exchanged between any two different surfaces at different temperature T_x and T_y can be determined by application of formula (33) for both considered surfaces which leads to the following formula:

$$B_{x-y} = B_x - B_y = \varphi_{x-y} A_x \varepsilon_{x-y} \frac{\sigma}{3}[3(T_x^4 - T_y^4) - 4T_0(T_x^3 - T_y^3)] \quad (34)$$

where $\varepsilon_{x\text{-}y}$ is the effective emissivity depending on emissivities ε_x and ε_y of respective surfaces and calculated like for radiation energy exchange. The effective emissivity simplifies to $\varepsilon_{x\text{-}y} = 1$ when the emissivities $\varepsilon_x = \varepsilon_y = 1$. Formula (34) is used appropriately for calculations of the five radiation exergies: $B_{f\text{-}d}$, $B_{d\text{-}sky}$, $B_{d\text{-}ch}$, $B_{ch\text{-}sky}$, and $B_{ch\text{-}gr}$.

The physical exergy of air (B_{a1}, B_{a2} and B_{a3}) is calculated from the common formula:

$$B_a = m\left[c_p(T_a - T_0) - T_0\left(c_p \ln\frac{T_a}{T_0} - R\ln\frac{p}{p_0}\right)\right] \quad (35)$$

where c_p and R are the specific heat and individual gas constant. Obviously, exergy of air entering the collector is zero, ($B_{a0} = 0$), because air is taken from environment.

Exergy B of convective heat transferred from a surface at temperature T to air (environmental or heated) is calculated, based on the energy E of this heat, from common formula:

Table 1b
Responsive trends of some output parameters to change of some input parameters (Study II)

#	Quantity	Units	Ref. value	Mono-variant changes of input parameters and resulting output	
1	2	3	4	9	10
2	*Input*				
3	S	W/m^2	**800**	**850**	800
4	H_3	m	**195**	195	195
5	D_f	m	**240**	240	**250**
6	He	m	**0.3**	**0.35**	**0.35**
7	*Output*				
8	p1	Pa	101,233.66	101,232.6	101,232.3
9	p2	Pa	99,686.13	99,685.76	99,685.68
10	p3	Pa	98,912	98,912	98,912
11	w1	m/s	1.10	1.47	1.54
12	m	kg/s	276	438	480
13	TfE	K	388.3	388.5	383.2
14	TdE	K	329.8	329.2	326.1
15	TaE	K	303.18	300.45	299.97
16	Ta1	K	318.19	312.75	311.79
17	Tch	K	292.43	292.30	292.06
18	*Energy*				
19	ea1	%	22.99	28.17	29.02
20	ea2	%	22.24	27.07	27.84
21	ea3	%	20.75	25.55	26.33
22	ew1	%	4.63e−4	1.24e−3	1.46e−3
23	ew2	%	1.93e−4	5.18e−4	6.10e−4
24	ew3	%	3.88e−4	1.04e−3	1.23e−3
25	ep1	%	0.3994	0.3975	0.3924
26	ep2	%	0.5118	0.5364	0.5357
27	ep3	%	0.5278	0.5802	0.5846
28	efa	%	17.81	21.53	22.38
29	efd	%	77.19	73.47	72.62
30	eda	%	5.577	7.040	7.032
31	ed0	%	26.05	24.16	23.74
32	edsky	%	44.21	40.92	40.59
33	edch	%	1.35	1.35	1.26
34	ech0	%	0.90	0.89	0.83
35	echsky	%	1.70	1.71	1.69
36	echgr	%|	0.230	0.227	0.209
37	EP	MW	229	368	404
38	eP (η_e)	%	0.64	0.96	1.03
39	*Exergy*				
40	ba1	%	1.2443	1.2614	1.2506
41	ba2	%	0.08	−0.45	−0.58
42	ba3	%	−0.61	−1.40	−1.58
43	bw1	%	5.148e−4	1.38e−3	1.62e−3
44	bw2	%	2.15e−4	5.76e−4	6.77e−4
45	bw3	%	4.31e−6	1.16e−5	1.37e−5
46	bp1	%	0.4438	0.4416	0.4360
47	bp2	%	0.5687	0.5960	0.5952
48	bp3	%	0.00586	0.00645	0.00650
49	bfa	%	5.10	6.18	6.17
50	bfd	%	17.24	16.38	15.42
51	bda	%	0.783	0.98	0.91
52	bd0	%	3.657	3.349	3.071
53	bdsky	%	2.237	2.022	1.756
54	bdch	%	0.114	.112	0.098
55	bch0	%	0.0145	0.0139	0.0123
56	bchsky	%	−0.044	−0.045	−0.046
57	bchgr	%	0.182	0.178	0.143
58	Δbf	%	72.66	77.44	78.41
59	Δba	%	4.20	5.45	5.39
60	Δbd	%	10.45	9.91	9.58
61	ΔbT	%	0.33	0.49	0.53
62	Δbch	%	1.21	1.50	1.58
63	bP (η_b)	%	0.70	1.07	1.15
64	*Ezergy*				
65	za1	%	8.96	10.62	10.87
66	za2	%	8.82	10.38	10.61
67	za3	%	8.24	9.84	10.08
68	zGa	%	7.28	8.92	9.19
69	zGT	%	0.89	1.32	1.41
70	zGch	%	0.673	1.00	1.06

$$B = E\left(1 - \frac{T_0}{T}\right) \tag{36}$$

Formula (36) is used appropriately for calculations of the four exergies $B_{f\text{-}a}$, $B_{d\text{-}a}$, $B_{d\text{-}0}$, and $B_{ch\text{-}0}$. Potential exergies of air are equal potential energies ($B_{p1} = E_{p1}$, $B_{p2} = E_{p2}$ and $B_{p3} = E_{p3}$). Kinetic exergies of air are equal kinetic energies, ($B_{w1} = E_{w1}$, $B_{w2} = E_{w2}$ and $B_{w3} = E_{w3}$).

The column 4 results (Table 1a) concerning exergy are used for the flow exergy diagram (Fig. 4). The diagram shows of how the solar radiation exergy arriving at the deck $B_S = 32.41$ MW, reduced by the 5% reflection, is distributed between five SCPP components; collector air, floor, deck, turbine and chimney. In the diagram the exergy streams B, W, are represented by their percentage values b related to the solar radiation exergy B_S. Exergy considerations disclose large degradation of solar radiation. The floor fully absorbs the received high temperature radiation exergy and converts it to the exergy at the lower temperature T_{fE}. Part of this T_{fE} exergy ($b_{f\text{-}d} = 17.24\%$) radiates to the deck and another part $b_{f\text{-}a} = 5.10\%$ is transferred by convection to heated air in the collector. The remaining large part ($\Delta b_f = 72.16\%$) is lost during irreversible processes of absorption and emission at the floor surface.

The power B_p performed by turbine is the same like in energy balance: $B_P = E_P = 0.23$ MW (exergy of work is equal to the work). Percentage power of turbine $b_P = 0.70\%$ represents the exergy efficiency η_b of the SCPP. Exergy efficiency is slightly higher because the same power is related to the radiation exergy which is smaller than the radiation energy. The exhausted exergy of air from chimney is negative $b_{a3} = -0.61\%$ whereas the exhausted potential and kinetic exergies are small; $b_{p3} = 0.01\%$ and $b_{w3} = e_{w3}$, respectively. The SCPP losses the exergy due to irreversibility and by radiation and convection heat transferred from deck and chimney to the sky and environment. Solar exergy reflected from the deck is $b_R = e_R = 5.00\%$.

The negative value of physical exergy (b_a) of air can be commented as follows. Exergy is a measure of departure of matter parameters from the equilibrium state with human environment. Therefore exergy should be always positive whenever the parameters are different from environment parameters. The exergy of matter within the sys-

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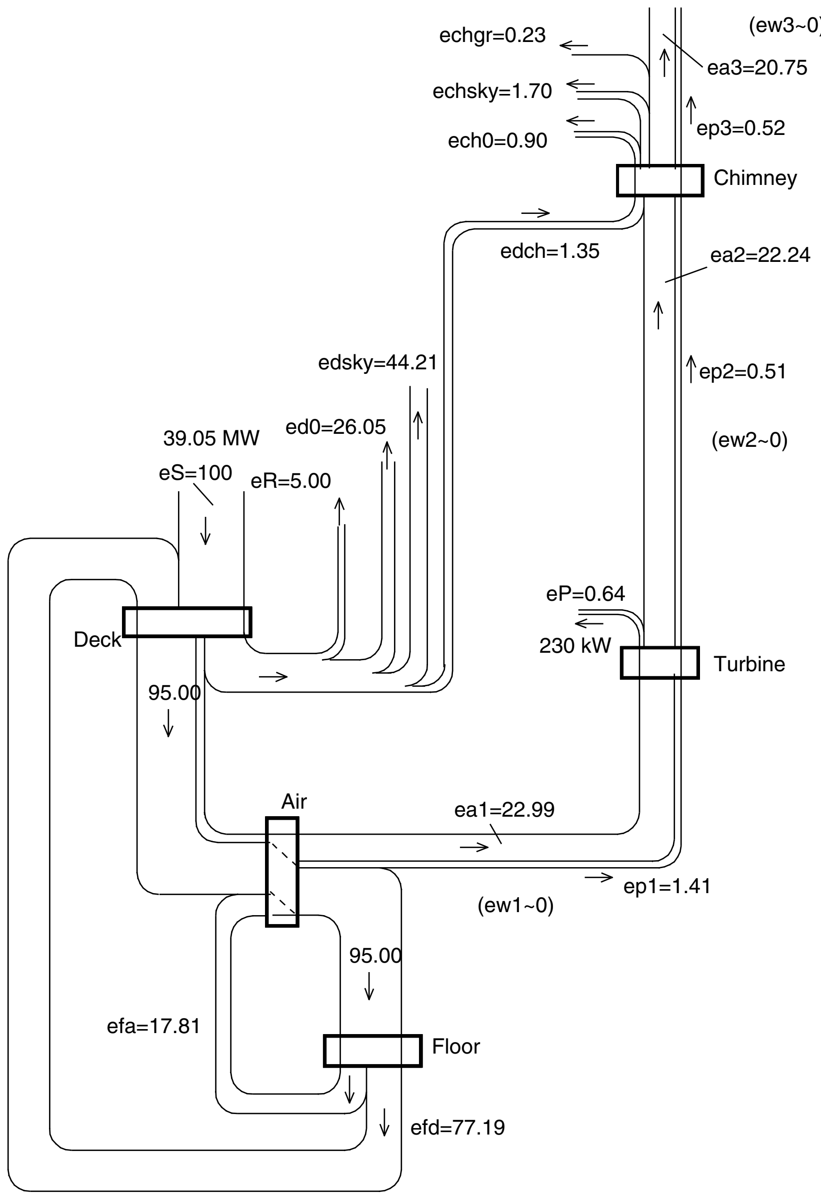

Fig. 3. ENergy balance of the SCPP according to Table 1, column 4 (the values are expressed in %).

tem is really always positive ("exergy of internal energy") whereas exergy of matter exchanged with the system ("exergy of enthalpy") can be negative for certain combination of matter parameters. Especially for air, this can happen, e.g., when, at the air temperature not much higher relative to environment, the air pressure is lower than atmospheric. The case of negative exergy is discussed with more details, e.g., by Petela (2008b).

5. Exergy analysis with using mechanical exergy component for substance

Another interpretation of processes is possible by application of the mechanical exergy concept which is one of the exergy components of substance. The mechanical exergy, proposed by Petela (2008b), takes into consideration the theoretical possibility that the exergy (maximum possible work) of a substance could be theoretically executed not necessarily at the earth surface level but eventually at the different environment prevailing at certain altitude to which the substance could be brought by the buoyancy effect. This effect occurs when density ρ of considered substance differs from density ρ_0 of environment ($\rho \neq \rho_0$). If mechanical exergy is applied then, on the input side of exergy balance equation, the gravity input term is introduced. Without such introduction the equation would be not completed. Thus the gravity input is calculated from exergy balance equation in which mechanical exergy of substance is used. In such equation the exergy loss is the same like in the traditional exergy balance, i.e. calculated from the Guoy–Stodola law.

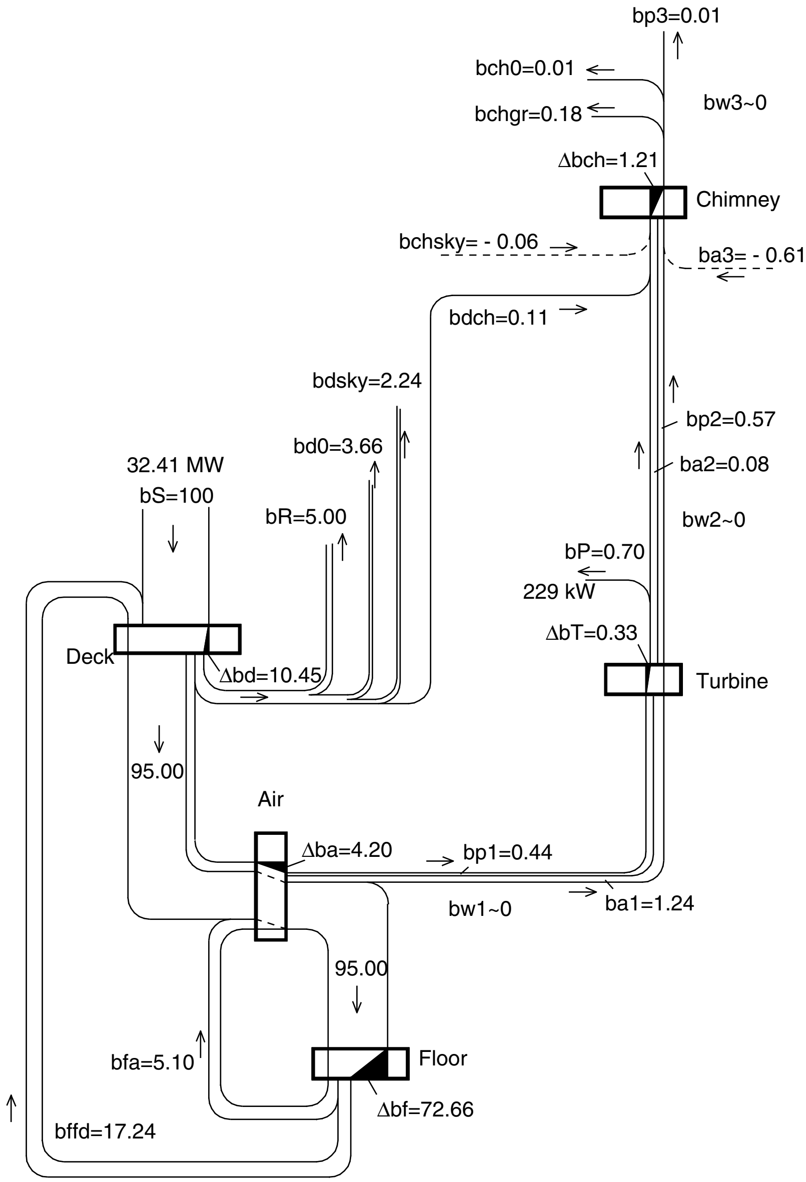

Fig. 4. EXergy balance of the SCPP according to Table 1, column 8 (the values are expressed in %).

It is supposed that the value of the gravity input (negative, positive or zero) is expressing the effect of gravitational field on the processes in which any substance takes a part.

In the SCPP the five components are considered (floor, deck, air in collector, turbine and chimney). Exergy balance equations for floor and deck do not contain terms for substance. These two equations contain only terms of the radiation and convection heat for which gravitational effect have been not considered so far. Therefore, the exergy balances for floor and deck remain here unchanged. However, the three modified exergy balance equations are considered for heating air in collector, turbine and chimney.

It is proposed to call the mechanical exergy of substance an "ezergy", denoted by Z, W, or z, %, to easier distinguish from the traditional exergy (B or b) used in Section 4. (The only substance considered in the SCPP is air.) Denotations of other exergy magnitudes remain in the present section unchanged because their values are unchanged. However, ezergy of air generally differs from exergy of air; $Z_a \geqslant B_a$, which results from definition of mechanical exergy (ezergy):

$$Z_a = \max(B_p + B_H, B_a) \tag{37}$$

where B_p is the potential exergy ($B_p = E_p$) B_a is the traditional physical exergy of air calculated from formula (35). Magnitude B_H is the physical exergy calculated also based on Eq. (35), however, for the environment parameters (temperature T_H and pressure p_H) prevailing at the altitude H:

$$B_H = m\left[c_p(T_a - T_H) - T_H\left(c_p \ln\frac{T_a}{T_H} - R\ln\frac{p}{p_H}\right)\right] \tag{38}$$

Petela (2008c) proposes the approximations of atmospheric air density varying with the altitude:

$$H = 1.215485 \times 10^6 - 1.214 \times 10^6 \rho_a^{6.02353 \times 10^{-3}} \quad (39)$$

and of approximated atmospheric parameters at altitude H:

$$T_H = 288.16 - 0.0093H + 3.2739 \times 10^{-7}H^2 - 2.9861 \times 10^{-12}H^3 \quad (40)$$

$$p_H = 101235\,e^{1.322 \times 10^{-4}H} \quad (41)$$

Except three Eqs. (30)–(32), Eqs. (28)–(36), from exergy consideration in Section 4, remain valid in this section. Eqs. (24)–(26) change respectively as follows:

$$G_a + B_{f\text{-}a} = Z_{1a} + B_{w1} + B_{a\text{-}d} + \Delta B_a \quad (42)$$

$$G_T + Z_{1a} + B_{w1} = Z_{2a} + B_{w2} + B_P + \Delta B_T \quad (43)$$

$$G_{ch} + Z_{2a} + B_{w2} + B_{f\text{-}ch} = Z_{3a} + B_{w3} + B_{cv} + B_{ch\text{-}0} + \Delta B_{ch} \quad (44)$$

where G_a, G_T and G_{ch} are the gravity inputs in ezergy balance for the collector air, turbine and chimney, respectively. Note that in ezergy balance equation the potential exergy does not appear as a separate term because this exergy is interpreted by ezergy of substance.

The column 4 results (Table 1a), concerning exergy and ezergy, are used for the flow ezergy diagram (Fig. 5). Again the diagram shows of how the solar radiation exergy $B_S = Z_S = 32.41$ MW, reduced by 5% reflection, arriving at the deck is distributed between five SCPP components; collector air, floor, deck, turbine and chimney in the case of using substance ezergy. In the diagram (Fig. 5) the ezergy streams Z, W, are represented by their percentage values z, related to the solar radiation exergy Z_S. The part of diagram (Fig. 5) related to floor and deck is the same like in Fig. 4, because substance does not appear in balances of the floor and deck. Also degradations of solar radiation and convective heat are the same like shown in Fig. 4

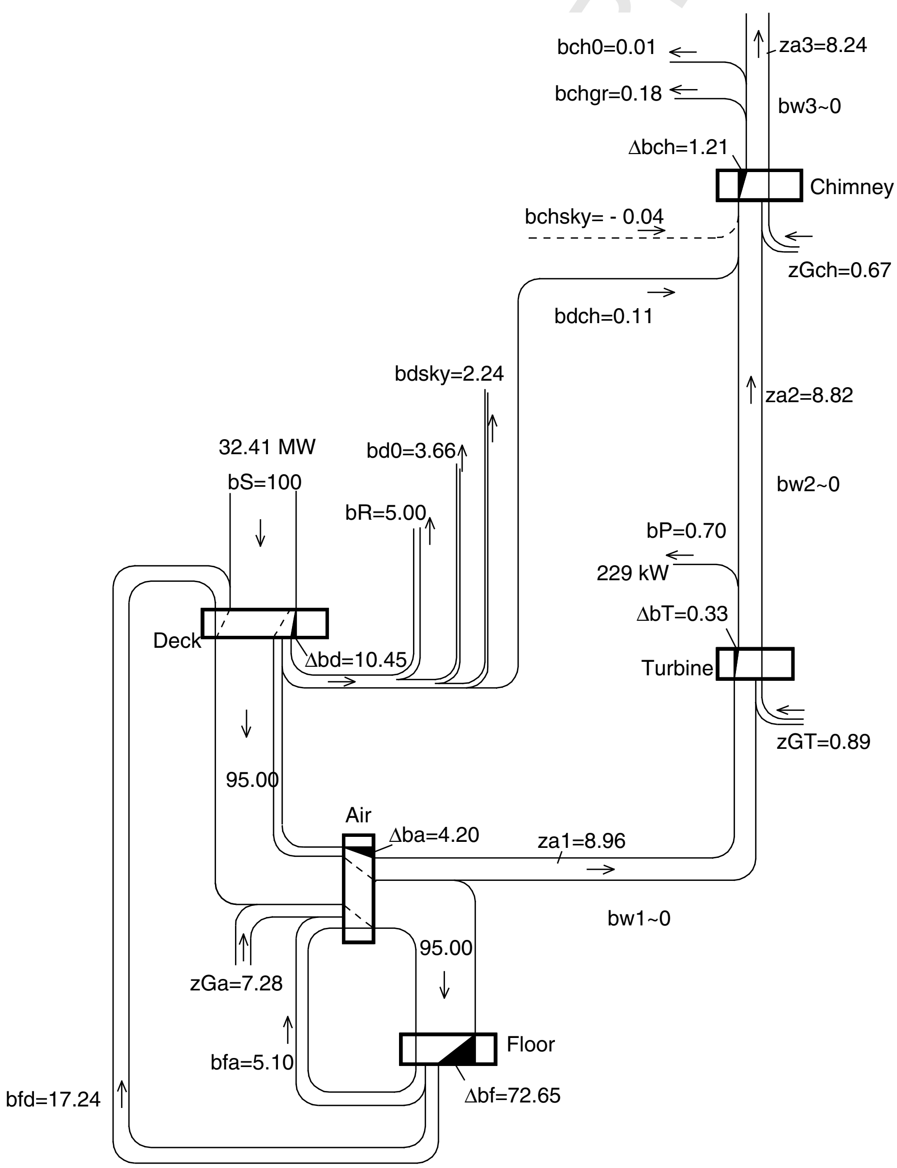

Fig. 5. EZergy balance of the SCPP according to Table 1, column 8 (the values are expressed in %).

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and also the power performed by turbine is unchanged (0.23 MW). Percentage power of turbine $z_P = b_P = 0.70\%$ represents the ezergy efficiency of SCPP. Specificity of the diagram (Fig. 5) is showing the relatively large ezergies of air $z_{a1} = 8.96\%$, $z_{a2} = 8.82\%$ and $z_{a3} = 8.24\%$. In result, the gravity inputs are $z_{Ga} = 7.28\%$ for the air in collector, smaller for turbine $z_{GT} = 0.89\%$ and the smallest for chimney $z_{Gch} = 0.67\%$

6. Responsive trends to the varying input parameters

The derived mathematical model of the SCPP responds to the input data as shown by the all computation results inTables 1a and 1b. The results can be used for illustration of the trends of the output data in response to changes in some input parameters. The values in column 4 of Table 1a (Study I) are considered as the reference values for studying the influence of the varying input parameters on the output data. Therefore, each of the next columns (5–8) corresponds to the case in which the input is changed only by the value shown in a particular column (bold case), whereas the other input parameters remain at the reference level.

For example column 7 corresponds to a change in the floor diameter D_f, which increases from 240 to 250 m. The 10 m D_f increase causes the 1 K increase of the effective temperature T_{fE} of floor from 388.3 to 389.3 K. At the same time:

w_1 decreases from 1.10 to 1.03 m/s
m decreases from 276 to 268 kg/s
E_P decreases from 229 to 222 kW
b_{a3} increases from −0.61 to −0.44%
b_{p3} decreases from 0.00568% to 0.00576%
z_{Ga} decreases from 7.28% to 6.89%, etc.

The model responses can be also the basis for deducing of many other observations. For example, it results from Table 1a, that increased solar radiation S from 800 (column 4) to 850 W/m^2 K (column 2) causes seemingly unexpected decrease in energy efficiency from 0.64% to 0.53%. However it can be deduced that increased S requires adjusting in the SCPP dimensions to better utilize the increased S. Thus it results from Table 1b (Study II) that if with growing S (from 800 to 850), at the same time the air entrance height H_e is adjusted, e.g., from 0.3 (column 4) to 0.35 (column 9), then the energy efficiency grows much higher (reaches 0.96%) in comparison to the case of growing H_e to the same value like in column 8 but at the unchanged $S = 800$ W/m^2 K. Similar reasoning about adjusting of the SCPP dimensions to the input situation can be also carried out for the increasing floor diameter D_f. The reasoning for both input (S and D_f) is illustrated by Table 2.

There could be also noticed that seemingly unexpected is the negative exergy of radiation exchanged between chimney and sky ($b_{ch\text{-}sky}$). As exergy of radiation at temperature below T_0 is positive, this effect, possible for disclosing only by exergy, occurs because T_{sky} is more below T_0 than T_{ch} is above T_0.

Generally, as a consequence of the many assumptions, the calculated quantitative responses to the varying SCPP input are of a limited certainty. However, perhaps better certainty can be expected for the determination of direction trends found in response to the varying input parameters. These trends result from Table 1a as the values and algebraic signs of the partial derivative $\partial M/\partial N$ interpreted by the finite differences:

$$\frac{\partial M}{\partial N} \approx \frac{\Delta M}{\Delta N} = \frac{M(i_M, j_M = 4) - M(i_M, j_N)}{N(i_N, j_N = 4) - N(i_N, j_N)} \qquad (45)$$

where M and N are any output and input variables, respectively, $i_M = 8, \ldots, 70$ is a row number from column 1; $i_N = 3, \ldots, 6$ is a row number from column 1; $j_M = 4, \ldots, 8$ is a column number from row 1; and $j_N = 4, \ldots, 8$ is the column number from row 1.

For example, consider direction trend of change in power E_P, (M), in response to change of diameter D_f, (N). Substituting to formula (45) respectively for $i_M = 37$, $i_N = 4$ and $j_N = 7$, $\partial M/\partial N = \partial E_P/\partial D_f = (229 - 222)/(0.3 - 0.35) = -140$ kW/m, where sign minus means that power E_P decreases with growing diameter D_f.

7. Conclusions

The overall process of power generation in the SCPP is very complex and, up to date, rather only some selected aspects of the SCPP have been studied. The present study attempts to develop analysis of total SCPP process. The complexity of such thermodynamic object enforced many simplifying assumptions. This necessity of simplification

Table 2
Analysis of influence of S and D_f on the SCPP energy efficiency ($H_3 = 195$ m)

Quantity	$D_f = 240$ m			Quantity	$S = 800$ W/m^2 K		
Input				*Input*			
S (W/m^2 K)	800	850	850	D_f (m)	240	250	250
H_e (m)	0.3	0.3	0.35	H_e (m)	0.3	0.3	0.35
Output				*Output*			
e_P (%)	0.64	0.53	0.96	e_P (%)	0.64	0.57	1.03

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should be not discouraging because, e.g., the Carnot cycle and its efficiency was derived with far going simplifications and in spite of this has basic significance in thermodynamics.

It is not easy to prove, but it is supposed that the proposed model is enough informative in terms of determination of the effects of varying input parameters on the SCPP output parameters, especially determination of the trends of these effects.

The proposed model involves some magnitudes which although are not precisely determining real situation, (e.g., the effective temperature of surface or the average convective coefficients of heat transfer), they however have not be assumed constant and thus they have certain freedom to vary and response showing its approximate value and trend of variation.

The presented considerations introduce the following original elements:

– formulation of energy balance of total SCPP,
– pioneer application of exergy balance for interpretation of component processes,
– pioneer application of ezergy balance for estimation of effect of gravity,
– involving exchange of radiation energy and exergy between chimney and deck,
– distinguishing the energy, exergy and ezergy losses to the environment and sky,
– proposing the convective-radiative effective temperature concept for the surfaces.

The purpose of the present introductory study was not to perform any optimization, although the optimum ratio $r_T = 2/3/$ for the relative pressure drop in turbine was assumed. The purpose was only to outline the methodology of exergy analysis applied to the SCPP and develop possible different thermodynamic interpretations of processes occurring in the SCPP. Based on the energy and exergy balances the distribution of solar input between the SCPP components was determined. The applied concept of mechanical exergy of air (air ezergy) allowed for additional interpretation of the processes. Based on the ezergy of air the positive gravity input was determined for the all components in which air takes role. The discussion on the meaning, significance and interpretation of the concept of gravity input, which can be positive, negative or zero, remains still open.

Obviously, the presented considerations leave many problems to study in the future, preferable within more complex thermodynamic model of SCPP.

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